TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 132.

THE INCREASE IN DIMENSIONS OF AIRPLANES - WEIGHT, AREA, AND LOADING OF WINGS.

By E. Everling.

From Technische Berichte, Volume III, Part 2.

March, 1923.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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Synopsis.

The percentage of weight of the wing relative to the total weight of the airplane ($c_w = 100\ W_w/W\%$) and the weight of the wing per unit area (W_w, kg/m²) of actual airplanes are represented in their relation to the wing area (S m²) and to the wing loading (W/S, kg/m²) - the influence of the latter will also be dealt with theoretically.

It is concluded that:-

1. The weight per unit area increases slightly with the wing area.
2. The percentage weight of the wing increases at a greater rate.
3. The weight of the wing per unit area increases rather rapidly as the wing loading increases.
4. On the contrary, the percentage weight of the wing decreases much more rapidly.
5. There are considerable deviations from these laws, particularly that under (1), and the exceptions are not confined to special types of airplanes.

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* From Technische Berichte Vol. III, Part 2. (1918).

6. The results of the theoretical and practical considerations regarding the influence of the wing loading on the weight of the wing are in agreement.

This investigation supplements the previous report (T.B. Vol. II, No. 2) in that it partly explains the variations in the percentage weight of the wings, there considered, in their relation to the dimensions of the airplane, particularly, by showing the influence of the wing loading, and in that it deals further with the question of the weight of the wings per unit of area.

Variation in the Weight of the Wings.

When apportioning the weight of the airplane into its main component parts (T.B. Vol. II, Part 3, pp. 563 and 579), with other considerations it was ascertained that the combined weight of the structure (wings, tail unit, fuselage, landing gear etc.) varied with the dimensions as well as with the wing loading, but that the actual relations existing were not completely illustrative of the slight variation of the weight of the wings with the dimensions of the airplanes and with the wing loading, which was indicated only in the first report, (T.B. Vol. II, Part 2, p. 279) but which was clearly evident from the table giving details of actual airplanes included in that report.

In the following portion of this treatise, the percentage weight of the other parts of the wing structure will be dealt with and their relation to the size of the airplane will be demonstrated. The size of the airplane will again be indicated by the total flying weight (W kg).

Table I.

Distribution of Weight with loading and coefficients for various types of airplanes.

1	2	3	4	5	6	7	8	9	10
No.	Type of air-plane	No. in former* Table I.	Total flying weight W kg	Com-bined Wt. of struc-ture W_c kg	Wt. of wing W_w kg	Wt. of tail W_t kg	Wt. of fuse-lage W_f kg	Wt. of acces-sories W_a kg	Wt. of landing gear W_l kg
1	Ea	1	535	206	62	9	62	31	42
2	Da	2	578	174	62	10	50	22	30
3	Eb	4	604	181	70	10	44	27	30
4	Db	5	632	207	82	13	68	15	29
5	Dc	7	780	245	105	15	60	25	40
6	Dd	8	841	219	84	15	60	25	35
7	De	9	845	252	125	17	56	11	43
8	Df	11	887	254	113	16	65	20	40
9	Dg	12	901	292	125	32	55	30	50
10	Dh	13	908	256	99	15	72	27	43
11	SEVa	15	947	336	112	23	151	5	45
12	Di	16	954	300	123	16	87	28	46
13	BE(I)	18	990	350	154	34	82	36	44
14	Dk	19	1026	326	143	13	78	33	59
15	Ba	20	1040	331	146	16	61	62	46
16	BE(II)	21	1040	349	153	34	82	36	44
17	Ca	22	1071	273	110	25	60	40	38
18	Cb	24	1130	325	135	21	74	43	52
19	Bb	25	1218	517	211	23	100	114	69
20	Cc	26	1256	332	140	18	80	45	49
21	Cd	27	1286	360	146	16	70	60	68
22	Ce	28	1300	392	164	20	84	60	64
23	Cf	29	1315	409	167	18	100	70	54
24	Cg	30	1322	402	180	22	110	28	62
25	Ch	31	1335	369	163	21	84	45	56
26	Ci	32	1346	327	150	15	62	42	58
27	Ck	33	1346	394	150	22	131	29	62
28	Cl	34	1350	458	212	20	110	50	66
29	Cm	35	1353	403	156	17	160	16	54
30	Cn	36	1359	459	174	23	120	80	62
31	Co	37	1475	404	192	16	88	56	52
32	Cp	38	1520	430	171	19	118	68	54
33	Cq	39	1546	420	189	23	102	43	63
34	Cr	40	1561	462	192	26	122	54	68
35	Cs	41	1585	440	192	24	107	45	72

*See T.B. Vol. II, Part 8, pp. 568-571.

Table I (Cont)

Distribution of weight with loading and coefficients for various types of airplanes.

1	2	3	4	5	6	7	8	9	10
No.	Type of air-plane	No. in former* Table I	Total flying weight W kg	Combined Wt. of structure W_w kg	Wt. of wing W_W kg	Wt. of tail W_t kg	Wt. of fuse-lage W_f kg	Wt. of acces-sories W_a kg	Wt. of landing gear W_l kg
36	Ct	42	1618	459	188	22	145	42	62
37	Cu	43	1620	499	204	21	128	62	84
38	Cv	44	1639	423	196	32	88	36	71
39	Cw	45	1643	431	178	22	130	47	54
40	Cx	46	1668	444	200	32	105	42	65
41	Ja**	49	2003	635	401	49	81	28	76
42	Ga	50	2785	988	541	54	124	86	183
43	Gb	51	2975	742	330	42	90	170	110
44	Gc	52	3024	862	466	18	252	25	101
45	Gd	53	3033	1216	440	50	470	160	96
46	Ge	55	3250	1002	395	61	266	135	145
47	Gf	57	3415	842	400	42	110	160	130
48	Gg	58	3618	1010	430	60	370	40	110
49	Gh	59	3618	1092	496	21	339	68	168
50	Gi	60	3648	1122	506	21	359	68	168
51	Gk	61	3795	1188	638	46	182	132	190
52	Ra	63	10203	4638	2070	365	1270	140	793
53	Rb	64	11460	4950	2050	400	1450	250	800
54	Rc	66	12953	5350	2350	400	1450	250	900
55	Rd	67	13035	5223	2350	400	1450	123	900
56	Dra	(Dra)***	571		84				
57	Dl	--****	616		97				
58	Sopwith triplane	6	690		135				
59	Dm	--	697		86				
60	Dn	--	738		115				
61	Drb	--	745		98				
62	Do	--	763		106				
63	Dp	--	838		143				
64	Spad SVII	10	857		99				
65	Dq	(Db)	860		122				
66	Dr	14	923		140				

**Armor considered as useful load.

***Parentheses are taken from Table I of the first report, T.B.V. [illegible] No. 2, p. 286.

****Dashes indicate new types.

Table I (Contd.)

Distribution of weight with loading and coefficients for various types of airplanes.

1	2	3	4	5	6	7	8	9	10
No.	Type of airplane	No. in former* Table I	Total flying weight W kg	Combined Wt. of structure W_c kg	Wt. of wing W_w kg	Wt. of tail W_t kg	Wt. of fuselage W_f kg	Wt. of accessories W_a kg	Wt. of landing gear W_l kg
67	Bc	--	940		167				
68	Ds	--	950		120				
69	Dt	(Df)	961		126				
70	Cy	--	975		90				
71	Sopwith 2-seater	17	989		115				
72	Aa	--	1035		126				
73	Cz	--	1043		120				
74	Bd	(Ba)	1069		175				
75	Be	23	1081		175				
76	Bf	--	1150		212				
77	Cα	--	1222		190				
78	Cβ	--	1238		178				
79	Cγ	--	1264		183				
80	Cδ	--	1313		156				
81	Cε	(Cc)	1320		165				
82	Cζ	--	1344		153				
83	Cη	(Cd)	1349		228				
84	Cθ	--	1352		155				
85	Cι	(Cg)	1496		206				
86	Cκ	--	1630		180				
87	Cλ**	(Ck)	1642		202				
88	Cμ	(Cl)	1686		218				
89	Cν	47	1730		214				
90	Jb	(Ja)	1831		214				
91	Jc	48	1876		200				
92	Gl	(Ga)	2785		545				
93	Gm	54	3171		375				
94	Gn	56	3400		450				
95	Handley Page	62	5900		867				
96	Re	(Ra)	7560		1215				
97	Rf	(Rd)	10000		1300				

**Wing areas, etc., supersede previous data.

Table I (Contd.)

$$c_c = c_w + c_t + c_f + c_l$$

1	2	3	11	12	13	14	15	16	17	18
No.	Type of air-plane	No. in former* Table I	$c_c = 100 \frac{W_c}{W}$ %	c_w $100 \frac{W_w}{W}$ %	c_t $100 \frac{W_t}{W}$ %	c_f $100 \frac{W_f+W_a}{W}$ %	c_l $100 \frac{W_l}{W}$ %	Wing area S m²	Wing loading W/S kg/m²	Wing Wt. per unit area W_w/S kg/m²
1	Ea	1	38.5	11.7	1.7	17.3	7.8	14.0	38.2	4.46
2	Da	2	30.0	10.7	1.7	12.5	5.1	16.7	34.6	3.72
3	Eb	4	30.0	11.6	1.6	11.9	4.9	15.9	38.0	4.40
4	Db	5	32.7	13.0	2.1	13.1	4.5	14.2	44.5	5.78
5	Dc	7	31.4	13.4	2.0	10.9	5.1	23.6	33.1	4.45
6	Dd	8	26.0	10.0	1.8	10.1	4.1	23.6	35.6	3.56
7	De	9	29.8	14.8	2.1	7.9	5.0	20.5	41.2	6.09
8	Df	11	28.6	12.8	1.7	9.6	4.5	20.9	42.4	5.42
9	Dg	12	32.4	13.9	3.6	9.4	5.5	25.8	34.9	4.84
10	Dh	13	28.2	10.9	1.7	10.9	4.7	22.9	39.6	4.32
11	SEVa	15	35.5	11.8	2.4	16.6	4.7	22.8	41.5	4.92
12	Di	16	31.5	12.9	1.7	12.1	4.8	22.8	41.8	5.40
13	BE(I)	18	35.4	15.5	3.4	12.0	4.5	36.4	27.2	4.22
14	Dk	19	31.8	14.0	1.2	10.9	5.7	26.0	39.5	5.51
15	Ba	20	31.8	14.0	1.5	11.9	4.4	41.7	24.9	3.50
16	BE(II)	21	33.5	14.7	3.3	11.3	4.2	36.4	28.6	4.01
17	Ca	22	25.5	10.3	2.3	9.3	3.6	24.0	44.6	4.58
18	Cb	24	28.8	11.9	1.9	10.4	4.6	32.4	34.9	4.17
19	Bb	25	42.4	17.3	1.9	17.5	5.7	42.5	28.7	4.97
20	Cc	26	26.4	11.2	1.4	9.9	3.9	27.6	45.5	5.08
21	Cd	27	28.1	11.5	1.2	10.1	5.3	34.4	37.4	4.24
22	Ce	28	30.1	12.6	1.6	11.0	4.9	36.9	35.2	4.44
23	Cf	29	31.1	12.7	1.4	12.9	4.1	35.7	36.8	4.67
24	Cg	30	30.4	13.6	1.6	10.5	4.7	37.4	35.3	4.82
25	Ch	31	27.7	12.2	1.6	9.7	4.2	36.1	37.5	4.51
26	Ci	32	24.3	11.2	1.1	7.7	4.3	38.2	35.2	3.92
27	Ck	33	29.2	11.2	1.6	11.8	4.6	28.6	47.1	5.25
28	Cl	34	33.9	15.7	1.5	11.9	4.8	40.6	33.2	5.22
29	Cm	35	29.8	11.5	1.2	13.1	4.0	37.6	36.0	4.15
30	Cn	36	33.7	12.8	1.7	14.7	4.5	40.4	33.6	4.31
31	Co	37	27.4	13.1	1.1	9.7	3.5	38.6	38.2	4.99
32	Cp	38	28.2	11.2	1.2	12.3	3.5	34.1	44.6	5.01
33	Cq	39	27.2	12.2	1.5	9.4	4.1	42.8	36.1	4.41
34	Cr	40	29.6	12.3	1.7	11.2	4.4	41.8	37.3	4.59
35	Cs	41	27.7	12.1	1.5	9.6	4.5	43.4	36.5	4.43

* See T.B. Vol. II, Part 8, pp. 568–571.

Table I (Contd.)

$$c_c = c_w + c_t + c_f + c_l$$

1	2	3	11	12	13	14	15	16	17	18
[illegible] No.	Type of air-plane	No. in for-mer* Table I	$c_c = 100\ \frac{W_c}{W}$ %	$c_w = 100\ \frac{W_w}{W}$ %	$c_t = 100\ \frac{W_t}{W}$ %	$c_f = 100\ \frac{W_f+W_a}{W}$ %	$c_l = 100\ \frac{W_l}{W}$ %	Wing area S m^2	Wing load-ing W/S kg/m^2	Wing Wt. per unit area W_w/S kg/m^2
36	Ct	42	28.4	11.6	1.3	11.6	3.9	40.2[illegible]	40.2	4.68
37	Cu	43	30.8	12.6	1.3	11.7	5.2	38.2	42.4	5.34
38	Cv	44	25.9	12.0	2.0	7.6	4.3	42.7	38.4	4.59
39	Cw	45	26.2	10.8	1.3	10.8	3.3	34.0	48.3	5.23
40	Cx	46	26.6	12.0	1.9	8.8	3.9	42.7	39.1	4.68
41	Ja**	49	31.7	20.0	2.5	5.4	3.8	50.8	39.4	7.89
42	Ga	50	35.5	19.4	1.9	7.6	6.6	73.5	37.9	7.36
43	Gb	51	24.9	11.1	1.4	8.7	3.7	74.0	40.2	4.46
44	Gc	52	28.5	15.4	0.6	9.2	3.3	84.8	35.6	5.50
45	Gd	53	40.1	14.5	1.6	20.8	3.2	78.7	38.5	5.59
46	Ge	55	30.9	12.2	1.9	12.3	4.5	79.2	41.0	4.99
47	Gf	57	24.6	11.7	1.2	7.9	3.8	68.7	49.7	5.82
48	Gg	58	27.9	11.9	1.7	11.3	3.0	77.5	46.7	5.55
49	Gh	59	30.2	13.7	0.6	11.3	4.6	84.8	42.7	5.85
50	Gi	60	30.8	13.9	0.6	11.7	4.6	84.8	43.0	5.97
51	Gk	61	31.3	16.8	1.2	8.3	5.0	99.8	38.0	6.39
52	Ra	63	45.5	20.3	3.6	13.8	7.8	332.0	30.7	6.23
53	Rb	64	43.2	17.9	3.5	14.8	7.0	332.0	34.5	6.17
54	Rc	66	41.3	18.1	3.1	13.1	7.0	332.0	39.0	7.08
55	Rd	67	40.1	18.0	3.1	12.1	6.9	332.0	39.2	7.08
56	Dra	(Dra)***		14.8				17.5	32.6	4.83
57	Dl	--****		15.8				17.7	34.9	5.49
58	Sopwith triplane	6		19.6				22.0	31.4	6.14
59	Dm	--		12.3				18.7	37.4	4.60
60	Dn	--		15.6				17.1	43.2	6.72
61	Drb	--		13.2				17.2	43.4	5.70
62	Do	--		13.9				17.1	44.6	6.20
63	Dp	--		17.1				22.3	39.8	6.42
64	Spad SVII	10		11.5				20.5	41.8	4.83
65	Dq	(Db)		14.2				16.9	50.9	7.22
66	Dr	14		15.2				22.2	41.6	6.30

Table I (Contd.)

$$c_c = c_w + c_t + c_f + c_l$$

1	2	3	11	12	13	14	15	16	17	18
No.	Type of air-plane	No. in former* Table I	$c_c = 100\frac{W_c}{W}$ %	c_w $100\frac{W_w}{W}$ %	c_t $100\frac{W_t}{W}$ %	c_f $100\frac{W_f+W_a}{W}$ %	c_l $100\frac{W_l}{W}$ %	Wing area S m^2	Wing loading W/S kg/m^2	Wing Wt. per unit area W_w/S kg/m^2
67	Bc	--		17.8				32.2	29.2	5.19
68	Ds	--		12.6				20.9	45.5	5.74
69	Dt	(Df)		13.2				23.9	40.2	5.29
70	Cy	--		9.2				25.8	37.8	3.49
71	Sopwith 2-seater	17		11.6				31.6	31.3	3.62
72	Aa	--		12.2				30.2	34.3	4.18
73	Cz	--		11.5				23.4	44.6	5.14
74	Bd	(Ba)		16.4				40.6	26.3	4.31
75	Be	23		16.2				40.6	26.6	4.31
76	Bf	--		18.4				27.1	42.4	7.82
77	Cα	--		15.5				40.5	30.3	4.69
78	Cβ	--		14.4				37.5	33.0	4.74
79	Cγ	--		14.5				35.0	36.2	5.22
80	Cδ	--		11.9				34.4	38.2	4.54
81	Cε	(Cc)		12.5				35.7	37.0	4.62
82	Cζ	--		11.4				37.1	36.3	4.12
83	Cη	(Cd)		16.9				35.9	37.6	[illegible].34
84	Cθ	--		11.5				34.4	39.3	4.51
85	Cι	(Cg)		13.7				41.3	36.2	4.98
86	Cκ	--		11.0				34.0	48.0	5.30
87	Cλ**	(Ck)		12.3				34.0	48.3	5.94
88	Cμ	(Cl)		12.9				42.7	39.5	5.10
89	Cν	47		12.4				40.7	42.6	5.27
90	Jb	(Ja)		11.7				42.7	43.0	5.02
91	Jc	48		10.7				37.2	50.5	5.38
92	Gl	(Ga)		19.6				73.5	37.9	7.42
93	Gm	54		11.8				75.0	42.3	5.00
94	Gn	56		13.2				75.6	44.9	5.95
95	Handley Page	62		14.7				152	38.6	5.68
96	Re	(Ra)		16.1				233	32.4	5.21
97	Rf	(Rd)		13.0				264	37.9	4.93

As in T.B. Vol. II, Part 2, it was shown that the weight of the wing was approximately proportional to the flying weight, that is to say, that the percentage weight of the wings being practically constant, we can take the area of the supporting surface (S m²) as an indication of the size of the airplane, and the wing loading (W/S, kg/m²) as a relation between the size and the weight, and consider these values as independent variables.

The measure of the weight of the wing is again considered as the relation between the weight of the wing (W_W, kg) and the total flying weight, or the percentage weight of the wing component ($c_W = 100\ W_W/W\%$) and, also, from the point of view of the wing weight per unit area* (W_W/S, kg/m²) this being a criterion of the lightness of the construction.

Weight of the Wings in Actual Airplanes.

Data respecting these four values for a series of actual airplanes are given in columns 4, 5, 13, 16, 17, 18 of Table 1. They differ only very slightly from the values in the previous tables; in one place, however, several small errors have been rectified, although these did not affect the conclusions previously reached. The mutual relation between the two independent and the two dependent variables is represented graphically in Figs. 1-4.

In the first place, it should be noted that the wing loadings under consideration vary mainly between 25 and 50 kg/m², while the

* The expression "wing weight per unit area" is used in the same sense as the "wing loading" or "weight per HP".

areas of the wings (owing to the greater number of small airplanes of which data were obtainable and the marked differences in the dimensions of the various types) lie principally between 16 and 43 m^2, a few being about 80 m^2, and some about 330 m^2; but between these groups there are only a few isolated values and, therefore, wide gaps exist. The values plotted on the diagrams are also widely scattered, especially those relating to the smaller types. This appears to be due not only to the larger number of the smaller types but also to the fact that the larger types are more standardized.

Generally, both the percentage weight of the wings and the weight per unit area increase with increasing area, the former more than the latter, as the former varies from 12% in small airplanes to 18% in the large types, while the weight per unit area varies from 4 to 6 kg/m^2 only. An outstanding exception to these values is afforded by the J-type airplane, which was referred to in the first report on account of its extraordinarily heavy wings. It represents a new type possessing certain aerodynamic advantages; and it is now possible to reduce the weight of these wings considerably, as is shown by the C_z No.73 in Table 1, this airplane being similar in construction to the J-type airplane. The other particularly high values belong to older types. On the other hand, in a light C-machine the wings are only 9.2% of the total weight, while in the first report the lowest value tabulated was 11.1%, applying to a G-type airplane.

The tendency with increasing wing loading is partly in the opposite direction to the above; the percentage weight of the wing component again increases very considerably, but the weight of the wing per unit area decreases at a more rapid rate. The exceptions to this are, practically, confined to those types of airplanes referred to in the preceding paragraph.

To summarize, approximate average values may be laid down as follows:

Table II.

Average values of percentage weight of wing component and wing weight per unit area.

Wing loading kg/m²	Weight per unit area kg/m²	Percentage weight of wing unit. %
25	4	16
30	4	15
35	$4\frac{1}{2}$	14
40	5	13
45	$5\frac{1}{2}$	12
50	6	11

Conclusions.

The relations thus established are not unexpected, as a superficial examination of the data given would lead one to suppose that the weight of the wing will increase with increasing area (that is, with increasing span), and that, under certain conditions, the decrease of the wing loading which necessarily follows, will lighten the wing. As, in addition, the weight of the wing has now been represented as a percentage of the total weight and in its relation to the wing area, it was to be expected that, with increased wing loading, for reasons of strength, in the first place, the value W_w/S would increase, while the percentage weight of the wing component would decrease, because the weight of the wing increases in this case more slowly than the total weight.

These relations can be more accurately appreciated if we imagine an airplane with its linear dimensions doubled. First, let the wing loading remain unchanged. Then, in agreement with Lanchester's Theory as shown in the first report, the wing area and the total weight are quadrupled, the weight of the wings is increased to eight times their former value, and, as then, the rate of increase of the weight of the wings is as the total load raised to the power of 1.5, while the strength of the wings remains unchanged. Accordingly, the wing area and the total weight increase as the square; the weight of the wing, as the cube; and the wing weight per unit area, and also the percentage weight of the wing component, as the first power, of the span.

Consequently, the percentage weight of the wings and the wing weight per unit area must always increase more slowly than the increase in the square root of the wing area; that is, with increasing wing area the curves showing the relation between wing weight per unit area and the wing surface and the percentage wing weight and wing surface will assume a parabolic form, as shown in Figs. 1 and 2, with the S line as axis of the parabola, the curves rising from left to right and open to the right.

Now in the first report it was shown regarding Lanchester's conclusion, that the weight of the wing is proportional to the total weight raised to the power of $1\frac{1}{2}$, must be replaced by the condition based on actual practice; that the weight of the wing is approximately proportional to the total weight, since it averages about 14% of the total weight. Accordingly, as seen previously in column 11 of Table 1 in T.B. Vol. II, Part 2, p.286, the percentage weight of the wing component must be virtually constant, or only increase slightly with increasing size of the airplane. Further, the weight per unit area, also, with constant wing loading, has almost a constant value, or increases very slowly with an increase in the size of the airplane. Both these conclusions are in agreement with Figs. 1 and 2.

As to the relation between the percentage weight of the wing component and the weight per unit area and the wing loading (Figs. 3 and 4), it is possible to arrive at a similar conclusion with rather more certainty, since in this case the different types of airplanes are quite varied, and with the great differences in the construction represented, emphasize the divergence between Lanchester's Theory and actual practice.

Now let it be assumed that the wing loading of any airplane is increased, say, to double its first value by first increasing the total weight, and secondly by reducing the wing area. The total weight, however, must not be allowed to become proportional to the cube of the dimensions as in these circumstances the conditions in regard to strength are thereupon varied.

It is necessary here to give more attention to the forces and the cross sectional areas of the parts; to the bending and resistance moments; to the buckling length and the moments of inertia; and it must be ensured that the breaking stress remains constant, so that the same constructional materials may be used.

Further, as in practice we have to deal with rather small variations in the wing loading, varying between about 25 and 50 kg/m^2, (that is, the wing loading may be doubled) and as the percentage weight of the wing component only varies between 10 and 20%, the close distinction drawn in the first report, between the total weight and the load carried by the wings at the roots (approximately equal to $W - W_w$) may be neglected in considering the following calculations; and to compensate in some way for this, the parts of the airplane which are directly supported by the wings, such as, fuel tanks, part of the weight of the engine, etc., are not taken into account.

During the investigations, it should be remembered that the various parts of the wing structure are subject individually to different kinds of stress - to simple tension, simple buckling or pure bending, combined buckling and bending or surface tension, because each creates a different variation in the dimensions and weights. The relation between the weights of the various parts which are included in the wing structure to the total weight of the wings, is shown by the following data taken from an earlier article: (Everling and Gaule. Einzel - gewichte von Flugzeugflugeln - T.B. Vol. I, p. 298).

Table III.

Relation between the weight of the various component parts of the wing structure to the weight of wings.

Kind of stress	Parts	Proportion of strucural weight	Stress	(kg/cm²), for a Force F (kg) or Moment M (cmkg)
Pure tension or compression	Cables, Fittings*	0.10	F/S	S=Cross sectional area, cm^2
Buckling	Struts, including internal struts.	0.15	Fl^2/I,	l =Buckling length, cm
Bending	Ribs & leading & trailing edges	0.25	M/R	R=Moment of resistance, cm^3
Combined bending & buckling	Spars	0.30	$F/S+M/R$,	I=Moment of inertia, cm^4
Surface tension	Fabric, Fittings**	0.20	F/t^2,	t=Thickness, cm
Total	Combined weight of structure	1.00		

On this basis, the variation of the percentage weight of the wing and the weight per unit of area with the doubling of the wing loading will now be investigated, it being brought about, in the first place, by doubling the total load. This doubles the forces on the struts included in the structure and the bending moments on the spars, resulting from the greater load in the bays along

* Only part of the weight of the fittings; the remainder is divided among the other members as the fittings are sometimes under complex stresses.

** In buckling, the basis of comparison is not the actual stress but a numerical quantity which is inversely proportional to the factor of safety for a buckling load. (In the case of the ultimate tensile strength this quantity would correspond with the tension. The surface tension stress contains another factor, which is not considered here.)

the spars (and the displacement of the nodal points).

As, in accordance with the last column of Table III, the stresses on the spars are produced through lateral flexure due to buckling and bending, and as, for their safety when subject to buckling and bending loads, the ratio of the longitudinal forces to the cross sectional area plus the ratio of the maximum bending moment to the moment of resistance of the cross-section, is a measure of their breaking strength; and since the numerators are doubled, the denominators must, at least, be doubled also. This is the same when the cross-sectional area of the spars - and hence their weight - is doubled, for then the moments of resistance automatically increase in a higher ratio, namely, $2^{3/2}$ times, with a geometrically similar increase of the cross-section to twice the area.

In the same way, the cross-sectional area, and, therefore, the weights of tension members, - for example, the wire bracing, - must be doubled; while the ribs which are only subject to bending moment rendered a twofold increase in the moment of resistance of the cross-section necessary, and the subjection of the struts to a buckling load which also necessitates a like increase (twofold) of the moment of inertia of the section in both cases with a corresponding geometrically similar increase in the cross-sectional area - that is, increase in weight in the case of the ribs and the struts of $2^{2/3}$ and $2^{1/2}$ respectively.

Further, with twice the wing loading, the covering fabric must be $2^{1/2}$ times as thick, that is, the weight must be increased to $2^{1/2}$ times its original value.

Hence, the total increase in weight will be such that the final weight will lie somewhere between twice and $\sqrt{2}$ times the original weight. Thus, the coefficient of increased size and the proportional increase in the weights of the various parts of the wings, according to Table III, amounts to

$$2\,(0.01 + 0.30) + 2^{2/3} \times 0.25 + 2^{1/2}\,(0.15 + 0.20) =$$

$$= 0.80 + 0.40 + 0.49 = 1.69 = 2^{0.76} = 2^{3/4}$$

approximately.

If the wing loading were increased 1.5 times, instead of as above the result would be -

$$0.60 + 0.33 + 0.43 = 1.36 = 1.5^{0.45} = (1.5)^{3/4}$$

that is, virtually the same power.

Thus, by doubling the total load in order to double the wing loading, the percentage weight of the wings becomes 0.85 and the weight of the wings 1.7 times the original value.

Next, let the wing loading be increased by diminishing the wing area and the appertaining structure for example, by half. The span, chord, and thickness of the wing can thus be decreased in the ratio $1/\sqrt{2}$ and a bending load on the spars and ribs is multiplied by $\sqrt{2}$ per unit length.

The loading in the bays between the struts remains constant, as do also the longitudinal forces on all structural members subject to end loading; while the bending moments on the spars are decreased in the same ratio as the lengths of the bays, namely

$1/\sqrt{2}$. The moments of resistance of the spars may, therefore, be decreased; but since their section must remain constant their weight can only be decreased in the same ratio as the reduction in their length, namely, $1/\sqrt{2}$. The members under tension may also be lightened in the same ratio.

On the other hand, the members subjected to buckling, on account of the $1/\sqrt{2}$ reduction in their length, may retain the original factor of safety with only one-half the original moment of inertia. Hence, their cross sectional area and also their length may be decreased in the proportion of $1/\sqrt{2}$, and their weight will, therefore, be reduced by a half.

As regards the members subject to pure bending, the moment of resistance required is only $1/\sqrt{2}$ of that originally necessary so that the section can be reduced by $\left(\frac{1}{2}\right)^{1/3}$, and their weight will therefore be only $\left(\frac{1}{2}\right)^{5/6}$ or approximately only half the original value (as the ribs need not be placed so close together) on account of the length being reduced by $\left(\frac{1}{2}\right)^{1/2}$.

Finally the fabric covering can be reduced to half the weight, as the original thickness must be maintained, because it takes the same load on half the area.

Thus, according to Table III, the coefficient of reduction in weight is here:-

$$\frac{1}{\sqrt{2}}\,(0.10 + 0.30) + \frac{1}{2}(0.15 + 0.25 + 0.20) = 0.28 + 0.30$$

$$= 0.58 = 2^{-0.79} = 2^{-4/5}$$

If, instead of 2 we take 1.5 we shall obtain the following instead of the above

$$0.33 + 0.40 = 0.43 = 1.5^{-0.49} = 1.5^{-4/5}$$

That is, virtually, the same power.

Thus, by halving the wing area, in order to double the wing loading, the percentage weight of the wings becomes 0.6 and the weight of the wings 1.2 of the original value.

Table IV gives the result of this investigation.

The geometric mean of the two modifications, which corresponds to a double wing loading produced by a simultaneous increase of the total load and a decrease of the wing area, is also given. It has an approximate value of about $1/\sqrt{2}$ and $\sqrt{2}$ respectively.

Table IV.

Variation of the relative weight of the wing unit and the weight per unit of area.

With twice the wing loading	Relative weight of wing.	Weight per unit of area.
	Vary in the proportion:-	
By doubling the total load	0.85	1.69
By halving the wing area	0.58	1.17
Geometric mean of the two values	0.70	1.41
Empirical coefficient according to Table 2	$\frac{11}{16} = 0.7$	$\frac{6}{4} = 1.5$

The percentage weight of the wing, thus, decreases in practice in almost exactly the same proportion with an increase in the wing loading as that obtained by the rough calculation. The agreement in the case of the weight of the wing per unit area is also very good.

The establishment of the empirical coefficients from the available data is certainly somewhat arbitrary owing to the rather wide differences between the various types; and the same might be said of the choice of the geometric mean. However, the empirical coefficients lie, in any case, in practice, within the limits of the variations which result from doubling the load or halving the area.

The previous divergencies between theory and practice, as shown in the first report, give place here to comprehensive agreement, although this agreement, it is true, is somewhat obscured by the great diversity in the data from which it is derived, caused by the differences in the types and the construction embodied therein and other contributory causes.

V. Analysis of the Structure from the point of view of the Variation in the Weight of Components with the size of the Airplane.

Synopsis.

In considering the division of the combined weight in the case of a number of actual airplanes into its four component parts: the fuselage and accessories, the tail unit (stabilizer, fin, rudder and elevator) the wing unit and the landing gear (including the tail skid, the average percentage weight of each unit can be taken as 11, 2, 13, and 5% respectively of the total weight, and 36, 6, 43 and 15% respectively of the combined weight.

The weights of the various parts of the wing unit in the case of average types of airplanes, are strikingly similar. The percentage weight of the fuselage, however, slightly decreases as the dimensions of the airplane increase, and those of the tail unit and landing gear unit become particularly important in the case of the giant airplanes used for the investigation, partly owing to the conditions in regard to strength, stability, size and instruction which are specified.

Analysis of the Structure.

The previous analysis of the total loaded weight of the airplane (T.B. Vol. II, Part 3, pp. 563-579) as to combined engine and propeller unit weight and useful load (fuel and cargo) showed that the percentage of the combined weight of the structure (fuselage, wings, tail unit, landing gear, etc.) increased in a certain ratio with increasing dimensions of the airplane. On the other hand, it was shown in the first report (T.B. Vol.II, Part 2, p. 279) that the weight of the wing contributed only in a small degree to this increase, and the fourth report (first part of this report, Table 1) the variations in the weight of the wing were explained by the variation in the wing loading.

We must now carry the investigation further and ascertain to what extent the other component parts contribute to the increase of the combined weight. For this purpose, the combined structural weight which is always taken as a proportion of the total weight, is split up into its component parts, and their relation to the dimensions of the airplane considered first from the point of view of data relating to actual airplanes and then from general considerations.

The required data have been given already in Table 1 of this report. In columns 5 - 10 the structural weights are given and in columns 11 - 15 the corresponding values of the weights of the various components are shown as percentages of the total flying weight.

The structure is analyzed according to the construction specifications into the weights of fuselage W_f, supporting surface (wings) W_w, tail unit (fins, stabilizer, elevators and rudders) W_t, Landing gear unit (inclusive of tail skid) W_l, and fuselage accessories W_a. Since, in the construction specifications, the weight of most of the accessories is included in the weight of the fuselage, the proportionate values of the two have been added together forming the total $(W_f + W_a)$.

The Component Weights and their dependence upon the size of the Airplane.

In the diagram these percentage weights of the components of the structure weight are plotted as ordinates against the size of the airplane (which is represented there by the flying weight W) (Fig. 5). Their sum gives the percentage combined structural weight $c_c = 100\ W_c/W$. As the weight of the wings has already been thoroughly investigated, it is of little importance here, so that the order from bottom to top is

1) Tail unit (stabilizer, fin, rudder, elevator), $c_t = 100\ W_t/W$.

2) Fuselage, including accessories, $c_f = 100\ (W_f + W_a)$.

3) Wing component (wings and ailerons), $c_w = 100\ W_w/W$.

4) Landing gear (landing gear and tail skid), $c_l = 100\ W_l/W$.

so that the two values, the variations in the weight of which must also be followed from general considerations are the lowest and can be more clearly seen. The percentage weight of the tail unit is, in fact, very small, about 1.7%; and shows, with few exceptions, only slight variations from the mean value so that the percentage weight of the fuselage plotted above it, generally begin about the same level and the position of the circle denoting the upper limit indicates the weight of the fuselage.

Table V.

Average Values of the Various Units of Airplanes.

1	2	3	4		5	6	7	8	9
No.	Types of airplanes	No. of types	Flying weight		Percentage of flying weight				
			From kg	To kg	Combined structure c_c %	Wt. of wing c_w %	Wt. of tail c_t %	Wt. of fuselage c_f %	Wt. landing gear c_l %
1	Rotary engine single seater	4	535	632	33	12	2	14	5
2	Vertical engine single seater	9	780	1026	31	13	2	11	5
3	Light 2-seater	4	990	1218	36	15	3	13	5
4	C-type airplane	23	1071	1668	29	12	2	11	4
5	J-type airplane	1	2003	2003	32	20	3	5	4
6	G-type airplane	10	2785	3795	30	14	1	11	4
7	R-type airplane	4	10203	13035	42	19	3	13	7
8	Mean of all types	55	535	13035	31	13	2	11	5

Table V (Cont.)

Average Values of the Various Units of Airplanes.

1	2	3	10	11	12	13
No.	Types of airplanes	No. of types	Percentage of combined structural weight.			
			$100 \frac{c_w}{c_c}$ %	$100 \frac{c_t}{c_c}$ %	$100 \frac{c_f}{c_c}$ %	$100 \frac{c_l}{c_c}$ %
1	Rotary engine single seater	4	36	5	42	17
2	Vertical engine single seater	9	42	6	36	16
3	Light 2-seater	4	43	7	37	13
4	C-type airplane	23	43	5	37	15
5	J-type airplane	1	63	8	17	12
6	G-type airplane	10	46	4	36	14
7	R-type airplane	4	44	8	31	17
8	Mean of all types	55	43	6	36	15

It can be seen still more clearly in the diagram than in Table 1, that the variation in the values is comparatively small and it appears justifiable that the average values be taken for the percentage combined structural weight for the separate types of airplane (Table V). The relation between the weights of the components and the combined weight presents still greater uniformity than the average values; according to the table all component parts of the combined structure contribute in a similar manner to the increase in the structural weight.

The main exception is the tail unit, the weight of which is comparatively smaller in large airplanes than in smaller types; but which increases, however, in giant airplanes to about double the original value.

The percentage weight of the fuselage on the contrary decreases with increasing flying weight from 42% of the combined structural weight in light single-seaters, to 31% in giant airplanes, while in medium-sized airplanes it is about 36 - 37% excluding the J-type airplanes, the peculiarities of which have already been alluded to.

Finally, the percentage weight of the landing gear unit in relation to the total loaded weight reaches the maximum in giant airplanes; in all other types, on the contrary, the landing gear unit percentage is lighter, in comparison to the structural weight.

Conclusions.

In order, first of all, to appreciate the variations in the weight of the tail unit, it must be remembered that some W-airplanes, (namely, those having a σ value of 0.6, see Table 1) have a very long fuselage and a comparatively small tail unit, and that on the contrary, the tail units of the [illegible]pe airplanes which are larger in proportion to the wing [illegible], necessitate the special reinforcement of the structure, thus increasing the weight.

Here, as in the first report, according to Lanchester's Theory, we might also consider the relation between the weight of the tail unit, and the total weight, wing area and wing loading, by a detailed examination of the stresses produced in the various members, as was made in the case of the wing unit, but this is hardly worth while in view of the fact that the weight of the tail unit is, at most, only 3% of the total weight.

With comparatively heavy fuselages it would be an advantage to estimate the increase of weight, for instance, when all linear dimensions are doubled while the wing loading remains constant.

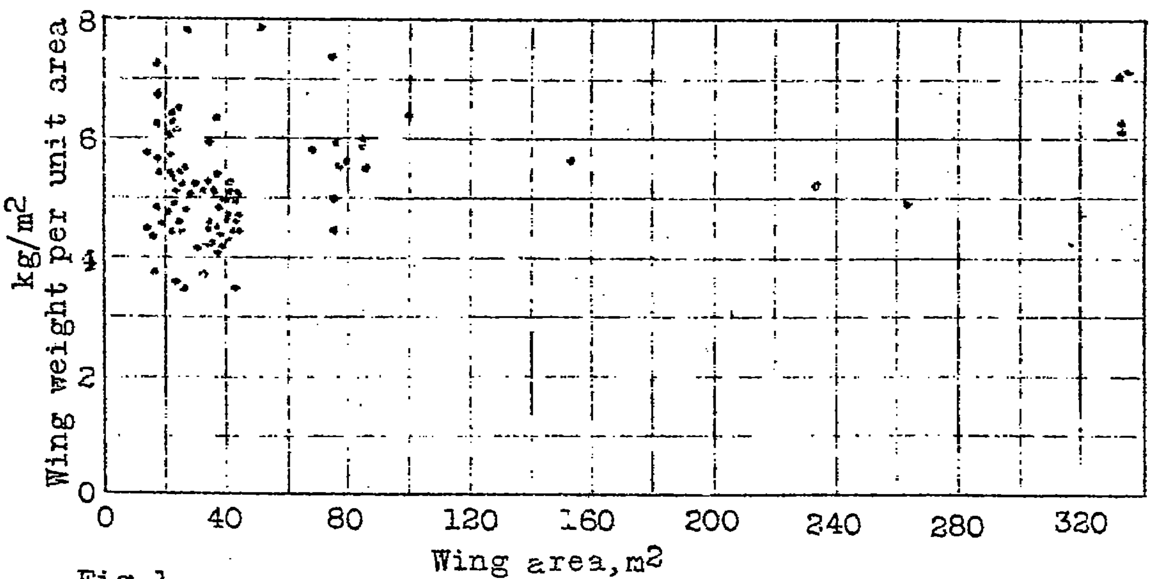
In that case, the wing area and the total weight are quadrupled, the righting moments due to the tail unit and transmitted through the fuselage, becomes eight times as great as do also the bending moments upon the fuselage (considered as a cantilever); the moment of resistance of the fuselage must therefore, be increased eight times, and this takes place automatically when all the linear dimensions are doubled. The weight of the fuselage, however, increases eight times, that is, as the total weight raised to the power of 1.5, exactly the same result as given in the first report for the weight of the wings. The influence of the weight of the fuselage itself upon the total load and on the bending moments may be neglected. Accordingly, the percentage weight of the fuselage must increase with increasing total weight as the square root of the total weight. In reality, however, the ratio of the weight of the fuselage to total weight remains constant, and the ratio to the structure weight is improved.

This can be explained as in the case of the weights of the wings, by the variation of the static requirements, in regard to local strength and the factor of safety in the structure, and further, owing to the possibility with a tail unit of larger area, and therefore, of greater weight, of constructing a shorter and thus lighter fuselage; or, in other words, owing to the fact that the assumption of a similar increase of dimensions, when applied to a different type of construction, no longer holds good.

Finally, it is obvious that in quite large airplanes, as also in light fast airplanes, the landing gear must be proportionately heavier, although in this case, the differences with various types of airplanes are small.

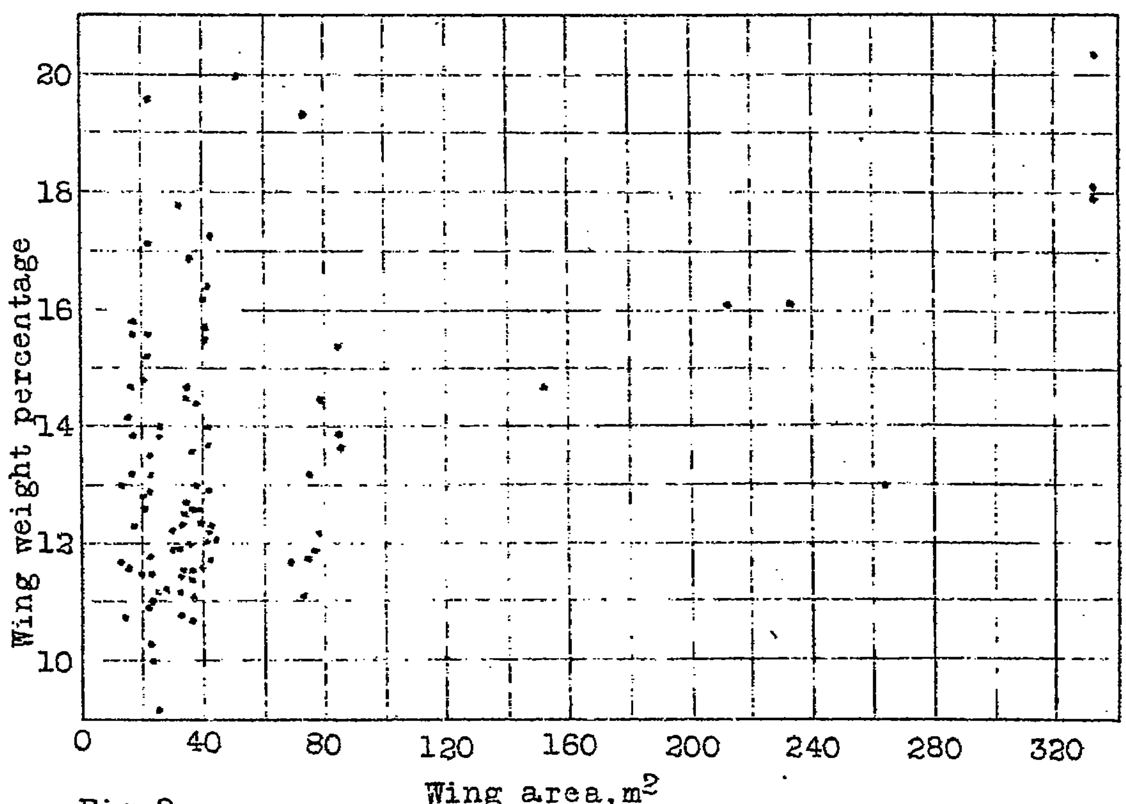
Thus, it is seen that the various components contribute fairly evenly to the percentage variations in the combined structural weight, which, as shown in the second report, in recent C- and G-airplanes, is less than 30%; but in R-type airplanes, is 40% of the total weight.

Translated by the National Advisory Committee for Aeronautics.



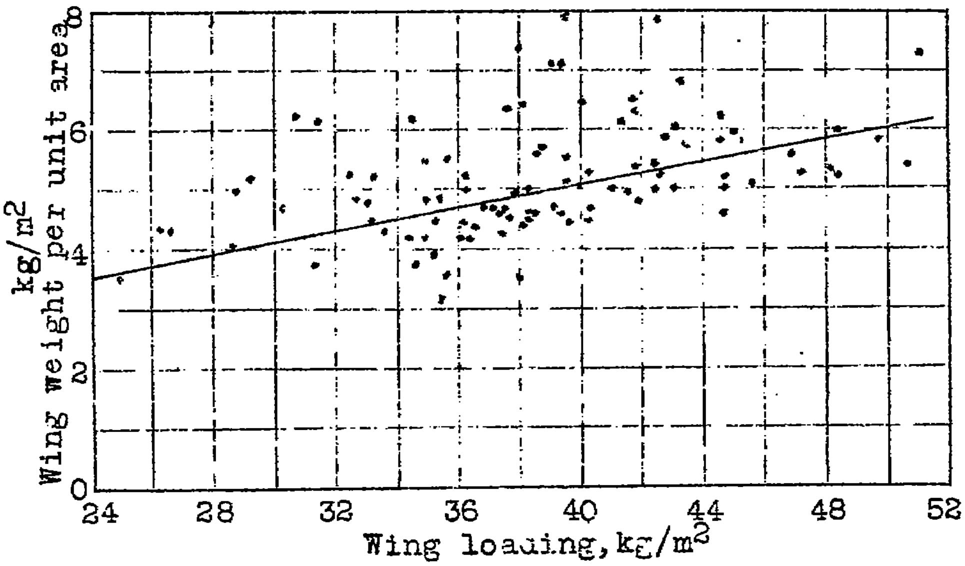
kg/m2
Wing weight per unit area
0
2
4
6
8
0
40
80
120
160
200
240
280
320
Wing area, m2

Fig.1



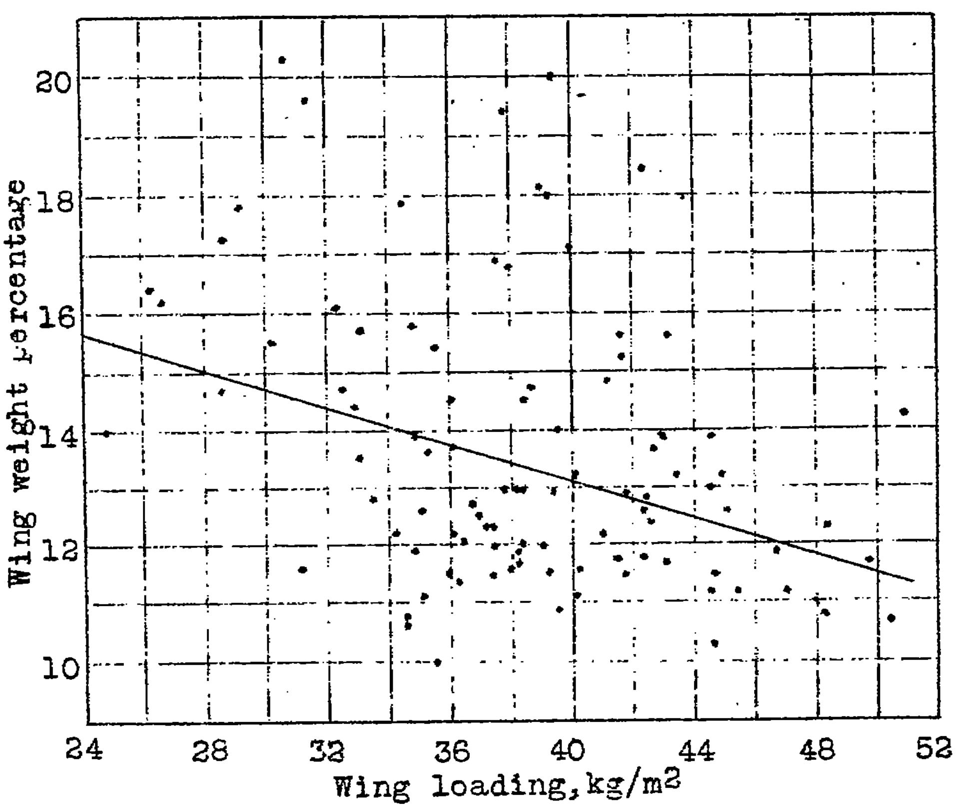
Wing weight percentage
10
12
14
16
18
20
0
40
80
120
160
200
240
280
320
Wing area, m2

Fig.2



Wing weight per unit area
kg/m2
0
2
4
6
8
24
28
32
36
40
44
48
52
Wing loading, kg/m2

Fig.3



Wing weight percentage
10
12
14
16
18
20
24
28
32
36
40
44
48
52
Wing loading, kg/m2

Fig.4

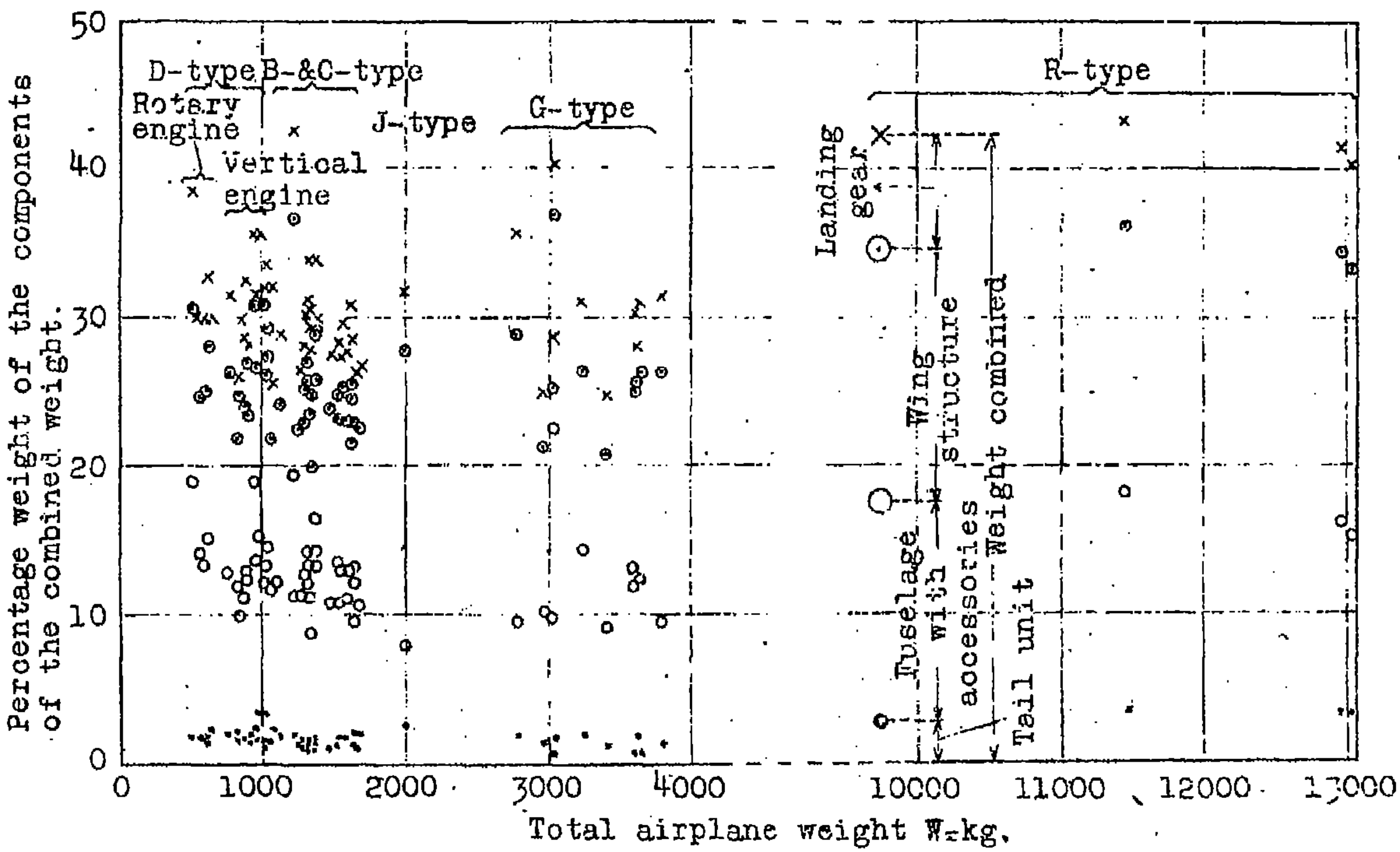


Fig.5 Analysis of component weights of the airplane in their relation to the combined weight.